COMMON P. G. ENTRANCE TEST – 2020

Test Booklet No. :

DEPT. OF HIGHER EDUCATION, GOVT. OF ODISHA

TEST BOOKLET

Subject Code **33** **Subject** **MATHEMATICS**

Time Allowed : ***90*** *Minutes* *Full Marks :* ***70***

: INSTRUCTIONS TO CANDIDATES :

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET **DOES NOT** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. You have to enter your **Hall Ticket No.** on the Test Booklet in the Box provided alongside. **DO NOT** write *anything else* on the Test Booklet.
3. **YOU ARE REQUIRED TO FILL UP & DARKEN HALL TICKET NO. & TEST BOOKLET NO. IN THE ANSWER SHEET AS WELL AS FILL UP TEST BOOKLET SERIAL NO. & ANSWER SHEET SERIAL NO. IN THE ATTENDANCE SHEET CAREFULLY. WRONGLY FILLED UP ANSWER SHEETS ARE LIABLE FOR REJECTION AT THE RISK OF THE CANDIDATE.**
4. **This Test Booklet contains 70 items (questions). Each item (question) comprises four responses (answers). You have to select the correct response (answer) which you want to mark (darken) on the Answer Sheet. In case, you feel that there is more than one correct response (answer), you should mark (darken) the response (answer) which you consider the best. In any case, choose ONLY ONE response (answer) for each item (question).**
5. You have to mark (darken) all your responses (answers) **ONLY** on the **separate Answer Sheet** provided by **using BALL POINT PEN (BLUE OR BLACK)**. See instructions in the Answer Sheet.
6. All items (questions) carry equal marks. All items (questions) are compulsory. Your total marks will depend only on the number of correct responses (answers) marked by you in the Answer Sheet. **There is no negative marking**.
7. **After you have completed filling in all your responses (answers) on the Answer Sheet and after conclusion of the examination, you should hand over to the Invigilator the *Answer Sheet* issued to you. You are allowed to take with you the candidate's copy / second page of the Answer Sheet along with the Test Booklet, after completion of the examination, for your reference.**
8. Sheets for rough work are appended in the Test Booklet at the end.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

1. If $A = \left\{-\frac{2n+3}{n} : n \in \mathbb{N}\right\}$, then :

(A) $\sup A = -2, \inf A = -5$

(B) $\sup A = -1, \inf A = -5$

(C) $\sup A = 0, \inf A = -5$

(D) None of these

2. Let f and g be bounded functions. Consider the statements :

(i) f o g is bounded.

(ii) f + g is unbounded.

Then which of the following is correct ?

(A) Both (i) and (ii) are true

(B) Both (i) and (ii) are false

(C) (i) is true but (ii) is false

(D) (i) is false but (ii) is true

3. Which one of the following is uncountable ?

(A) $\mathbb{N} \times \mathbb{N}$

(B) $\mathbb{R} \times \mathbb{R}$

(C) $\mathbb{Q} \times \mathbb{Q}$

(D) $\mathbb{Z} \times \mathbb{Z}$

4. The number of limit points of the set $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ is :

(A) 0

(B) 1

(C) 2

(D) 3

5. Let $\{a_n\}$ be a sequence of positive real numbers. Suppose that $l = \lim_{n \to \infty} \frac{a_{n+1}}{a_n}$.

Which of the following is true ?

(A) If $l = 1$, then $\lim_{n \to \infty} a_n = 1$

(B) If $l = 1$, then $\lim_{n \to \infty} a_n = 0$

(C) If $l < 1$, then $\lim_{n \to \infty} a_n = 1$

(D) If $l < 1$, then $\lim_{n \to \infty} a_n = 0$

6. Which one of the following is not true ?

(A) Every constant sequence is always bounded

(B) Every convergent sequence is always bounded

(C) Every Cauchy sequence is always convergent

(D) Every bounded sequence has always a convergent subsequence

7. $\sum_{k=1}^{\infty} \frac{1}{k^2 + k}$ is equal to :

(A) 0 (B) 1

(C) $\frac{1}{2}$ (D) 2

8. Suppose that $0 \le d_k \le a_k \le c_k$ for all $k \ge N$ for some $N \in \mathbb{N}$. Then :

(A) If $\sum_{k=1}^{\infty} c_k$ converges, then $\sum_{k=1}^{\infty} a_k$ also converges

(B) If $\sum_{k=1}^{\infty} a_k$ converges, then $\sum_{k=1}^{\infty} c_k$ also converges

(C) If $\sum_{k=1}^{\infty}$ If a_k diverges, then $\sum_{k=1}^{\infty} d_k$ also diverges

(D) If $\sum_{k=1}^{\infty}$ If c_k diverges, then $\sum_{k=1}^{\infty} d_k$ also diverges

9. Let $S_n = 1 + \frac{(-1)^n}{n}$, $n \in \mathbb{N}$. Then the sequence $\{S_n\}$ is :

(A) Monotonically increasing and is convergent to 1

(B) Monotonically decreasing and is convergent to 1

(c) Neither monotonically increasing nor monotonically decreasing but is convergent to 1

(d) Divergent

10. $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{k}{k^2 + n^2}$ equals to :

(A) 0 (B) $\log \sqrt{2}$

(C) $\log \sqrt{3}$ (D) $\log 2$

11. The series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^{\alpha}}$ for $\alpha > 1$ is :

(A) Uniformly convergent

(B) Absolutely convergent

(C) Both uniformly and absolutely convergent

(D) None of these

12. The radius of convergence of the power series $\sum_{n=0}^{\infty} 2^{2n} x^{n^2}$ is :

(A) $\frac{1}{4}$

(B) 1

(C) 2

(D) 4

13. Let $f(x) = \begin{cases} x+1, & x \geq 2 \\ 2x-1, & 1 < x < 2, \\ x-1 & x \leq 1 \end{cases}$

Then f(x) is discontinuous at :

(A) 1

(B) 2

(C) 3

(D) None of these

14. For what value of k, $f(x) = \begin{cases} \dfrac{\sqrt{7x+2}-\sqrt{6x+4}}{x-2}, & x \geq -\dfrac{2}{7} \text{ and } x \neq 2 \\ k, & x = 2 \end{cases}$,

is a continuous function ?

(A) $-\frac{2}{7}$

(B) $\frac{6}{7}$

(C) $\frac{1}{4}$

(D) $\frac{1}{8}$

15. $\text{Lim}_{x \to +\infty} \dfrac{\ln(l + e^x)}{1 + x}$ equals to :

(A) 0

(B) 1

(C) 2

(D) 3

16. If $f(x) = \sum_{n=0}^{\infty} 2^n x^n$, then $f^{(33)}(0)$ is equal to :

(A) 33!

(B) 2^{33}

(C) $33! \times 2^{33}$

(D) $\frac{33!}{2^{33}}$

17. The Taylor series for $f(x) = 2x^2 + 4x - 3$ about 1 is :

(A) $2(x-1)^2 + 4(x-1) - 3$ (B) $2(x-1)^2 + 8(x-1) - 3$

(C) $2(x-1)^2 - 8(x-1) + 3$ (D) $2(x-1)^2 + 8(x-1) + 3$

18. If z is implicitly defined as a function of x and y by $x^2 + y^2 + z^2 = 1$, then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is equal to :

(A) $\frac{1}{z} - z$ (B) $z - \frac{1}{z}$

(C) $z + \frac{1}{z}$ (D) $-z - \frac{1}{z}$

19. The maximum and minimum values of $f(x, y) = 1 - x^2 - y^2$ subject to constraint $x + y = 1$ with $x, y \geq 0$, respectively, are :

(A) 0 and 1

(B) $\frac{1}{2}$ and 0

(C) 1 and 0

(D) 0 and $\frac{1}{2}$

20. The equation of the tangent to the curve $3x^2 + 5y^2 = 23$ at (–1, 2) is :

(A) $6x + 10y = 0$

(B) $-3x + 10y + 23 = 0$

(C) $-6x + 10y = 0$

(D) $3x - 10y + 23 = 0$

21. The value of $\int_1^2 \int_3^4 (20 - xy)\, dy\, dx$ is :

(A) $\frac{59}{4}$ (B) $\frac{21}{4}$

(C) $\frac{9}{4}$ (D) $\frac{5}{4}$

22. The value of $\iiint_B z^2ye^x\,dV$, where B is the box given by $0 \le x \le 1$, $1 \le y \le 2$ and $-1 \le z \le 1$, is :

(A) $1 - e$ (B) $2 - e$

(C) $e - 1$ (D) $e - 2$

23. Let $A = \begin{bmatrix} 3 & 2 & 0 \\ 5 & a & 7 \\ 8 & 7 & b \end{bmatrix}$. If trace of A is 22, then $(a + b)^2$ is equal to :

(A) 121 (B) 216

(C) 361 (D) 400

24. If S and T are respectively, the set of all symmetric and skew-symmetric matrices of order n, then $S \cap T$ is :

(A) An empty set

(B) A singleton set

(C) Finite elements with n elements

(D) An infinite set

25. The determinant of the matrix $\begin{bmatrix} 2 & 5+3x & 8+6x+5x^2 \\ 2 & 5+3y & 8+6y+5y^2 \\ 2 & 5+3z & 8+6z+5z^2 \end{bmatrix}$ is equal to :

(A) $80(x-y)(y-z)(x-z)$

(B) $30(x-y)(y-z)(x-z)$

(C) $80(x-y)(y-z)(z-x)$

(D) $30(x-y)(y-z)(z-x)$

26. If A and B are two square matrices of order n, then which of the following is true ?

(A) $\text{rank}(A + B) = \text{rank}(A).\text{rank}(B)$

(B) $\text{rank}(A + B) = \text{rank}(A) + \text{rank}(B)$

(C) $\text{rank}(A + B) \ge \text{rank}(A) + \text{rank}(B)$

(D) $\text{rank}(A + B) \le \text{rank}(A) + \text{rank}(B)$

27. If $A = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$ and I is an identity matrix of order 2, then rank(I – A) is :

(A) 3
(B) 2
(C) 1
(D) 0

28. The system of equations : $x + 2y = 3$ and $3x + 6y = 4$ has :

(A) No solutions
(B) Unique solution
(C) Two solutions
(D) Infinite number of solutions

29. If $A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 1 & 0 \\ \frac{1}{5} & 0 & 0 \end{bmatrix}$, then one of the eigenvalues of $5I + A$ is :

(A) 5
(B) 4
(C) 3
(D) 2

30. The minimal polynomial associated with the matrix $A = \begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix}$ is :

(A) $(t + 3)(t - 6)$
(B) $(t + 3)$
(C) $(t - 6)$
(D) None of these

31. If V is the vector space of all real valued functions over R, then which of the following is not a subspace of V ?

(A) $W_1 = \{f : 3f''(x) + 2f(x) = 0\}$
(B) $W_2 = \{f : 3f''(x) - 2f(x) = 7\}$
(C) $W_3 = \{f : f(x) = f(-x)\}$
(D) $W_4 = \{f : f(x) = -f(-x)\}$

32. The dimension of the subspace $\{(x,y,z) : x - y + z = 0 \text{ and } 2x + 7y + 11z = 0\}$ of $\mathbb{R}^3 (\mathbb{R})$ is :

(A) 3
(B) 2
(C) 1
(D) 0

33. Which of the following transformations from $\mathbb{R}^3(\mathbb{R})$ to $\mathbb{R}^3(\mathbb{R})$ is a linear transformation?

(A) T(x, y, z) = (xy, 0, 0)

(B) T(x, y, z) = (x + y, y + z, z + 2x)

(C) T(x, y, z) = (x, cosy, z)

(D) T(x, y, z) = (x + y, y + z, z + 2)

34. Consider T : $\mathbb{R}^4(\mathbb{R}) \rightarrow \mathbb{R}^3(\mathbb{R})$ given by T(a, b, c, d) = (a + b, b + c, c + d). Then :

(A) rank(T) = 2, nullity(T) = 1

(B) rank(T) = 1, nullity(T) = 3

(C) rank(T) = 3, nullity(T) = 1

(D) rank(T) = 1, nullity(T) = 2

35. Consider the continuous functions f(x) = 2 + x and g(x) = x^2 defined in [0,1], then the value of the inner product $\langle f(x), g(x) \rangle$ is :

(A) $\frac{12}{13}$

(B) $\frac{13}{12}$

(C) $\frac{12}{11}$

(D) $\frac{11}{12}$

36. |z - 4i| - |z + 5i| = 6 is a/an :

(A) Circle

(B) Ellipse

(C) Hyperbola

(D) Straight line

37. The value of i^i is equal to :

(A) $e^{\frac{\pi}{2}}$

(B) $e^{\frac{-\pi}{2}}$

(C) $-e^{\frac{i\pi}{2}}$

(D) $e^{\frac{i\pi}{2}}$

38. The function f(z) = sin x cosh y + i cos x sinh y is :

(A) Both continuous and analytic

(B) Continuous

(C) Analyt

(D) None of these

39. Suppose A(1, – 2), B(– 3, 4) and C(2, 2) are the three vertices of a triangle ABC. Then the length of the median from C to the side AB is :

(A) $\sqrt{22}$

(B) $\sqrt{17}$

(C) $\sqrt{14}$

(D) $\sqrt{10}$

40. The value of $\oint_C \frac{e^{2z}}{(z+1)^4}\,dz$; where $C : |z| = 3$, is :

(A) $\frac{16\pi i}{3e^2}$

(B) $\frac{8\pi i}{3e^2}$

(C) $\frac{16\pi i}{e^2}$

(D) $\frac{8\pi i}{e^2}$

41. If $A = \{a, b, c, d\}$ and $B = \{1, 2, 3\}$, then total number of functions from A to B is :

(A) 12

(B) 24

(C) 64

(D) 81

42. If $f : X \to Y$ and A, B are subsets of Y, then which of the following is correct ?

(A) $f^{-1}(A \cup B) \subset f^{-1}(A) \cup f^{-1}(B)$

(B) $f^{-1}(A \cup B) \supset f^{-1}(A) \cup f^{-1}(B)$

(C) Both (A) and (B)

(D) None of these

43. For integers a, b and c, which of the following is not true ?

(A) If $a|b$ and $c|d$, then $ac|bd$

(B) If $a|bc$ with $\gcd(a, b) = 1$, then $a|c$

(C) If $a|b$ and $b \neq 0$, then $|a| \leq |b|$

(D) All of the above are incorrect

44. Suppose N is a normal subgroup of a group G. Then which of the following is true?

(A) If G is an infinite group, then G/N is an infinite group

(B) If G is an abelian group, then G/N is a cyclic group

(C) If G is a cyclic group, then G/N is an abelian group

(D) None of these

45. Consider the following two statements :

P : The set Z^+ under addition is a group.

Q : The set Z^+ under multiplication is a group.

Then :

(A) P is true

(B) Q is true

(C) Both P and Q are true

(D) Both P and Q are not true

46. If $P = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ are two permutations, then PQ is :

(A) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$

47. The set of all permutations on a set S of 5 elements forms a group of order :

(A) 6!

(B) 6

(C) 5

(D) 5!

48. The number of elements of order 6 in a cyclic group of order 24 is :

(A) 1

(B) 2

(C) 3

(D) 4

49. The alternating group A_4 has no subgroups of order :

(A) 2

(B) 3

(C) 4

(D) 6

50. The number of generators of the additive group $\mathbb{Z}_{36}$ is equal to :

(A) 6

(B) 12

(C) 18

(D) 36

51. Which of the following rings is not an integral domain ?

(A) $\mathbb{Z} \oplus \mathbb{Z}$

(B) $\mathbb{Z}_p$; p is prime

(C) $\mathbb{Z}[x]$

(D) $\mathbb{Z}\left[\sqrt{2}\right] = \left\{a + b\sqrt{2} : a, b \in \mathbb{Z}\right\}$

52. The ring of Gaussian integers $\mathbb{Z}[i]$ has characteristic :

(A) 0

(B) 2

(C) 4

(D) 6

53. The general solution of the differential equation $2xy\,dx + (x^2 + y^2)\,dy = 0$ is :

(A) $y^3 + 3x^2 y = k$

(B) $y^2 + 3x^2 y = k$

(C) $3y^3 + x^2 y = k$

(D) $3y^2 + x^2 y = k$

54. The solution of the differential equation $\frac{dy}{dx} + y \tan x = \sec x$ that passes through the point $(\pi, 2)$ is given by :

(A) $y = 2\sin x + \cos x$

(B) $y = 2\sin x - \cos x$

(C) $y = \sin x + 2\cos x$

(D) $y = \sin x - 2\cos x$

55. The general solution of the differential equation $y'' + 4y' + 4y = 0$ is $y =$:

(A) $c_1e^{-2x} - c_2xe^{-2x}$

(B) $c_1e^{2x} - c_2xe^{2x}$

(C) $c_1e^{-2x} + c_2xe^{-2x}$

(D) $c_1e^{2x} + c_2xe^{2x}$

56. The particular solution of $4y'' + 12y' + 9y = 0$; $y(0) = 3$ and $y'(0) = -2$ is $y = pe^{-\frac{3}{2}x} + qxe^{-\frac{3}{2}x}$, then :

(A) $p = \frac{5}{2}, q = 3$

(B) $p = 3, q = \frac{5}{2}$

(C) $p = \frac{5}{2}, q = \frac{1}{3}$

(D) $p = 3, q = \frac{2}{5}$

57. To convert the differential equation $y' = \frac{xy^2 - y}{x}$ into an exact differential equation we need the integrating factor :

(A) $\frac{1}{(xy)^2}$

(B) $-\frac{1}{(xy)^2}$

(C) $-\frac{1}{xy}$

(D) $\frac{1}{xy}$

58. Which of the following differential equation is not linear ?

(A) $y''' + xy' + y = x^2$

(B) $3y' + xy = \sin x$

(C) $y'' - y = 0$

(D) $y'' + yy' = x^2$

59. The Wronskian of the set {sin 3x, cos 3x} is :

(A) – 3

(B) – 2

(C) – 1

(D) 1

60. The degree and order of the differential equation $y'' = \sqrt{1 + y'}$, respectively, are :

(A) 1 and 2

(B) 2 and 1

(C) 2 and 2

(D) 1 and 1

61. If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$, then $\frac{\partial^2 u}{\partial x \partial y} = ?$

(A) $\frac{x^2 - y^2}{x^2 + y^2}$

(B) $\frac{x^2 + y^2}{x^2 - y^2}$

(C) $\frac{x - y}{x + y}$

(D) $\frac{x + y}{x - y}$

62. The equation $e^{2x}u_{xx} + 2e^{x+y}u_{xy} + e^{2y}u_{yy} = 0$ is :

(A) Hyperbolic

(B) Parabolic

(C) Elliptic

(D) Ultrahyperbolic

63. If for a real continuous function $f(x)$, $f(a)f(b) < 0$, then in $[a, b]$ for $f(x) = 0$, there is / are :

(A) Exactly one root

(B) No roots

(C) At least one root

(D) Exactly two roots always

64. The order of convergence of Regula-Falsi method is :

(A) 1.245

(B) 1.618

(C) 2.147

(D) 3.141

65. The Newton-Raphson method formula for finding the square root of a real number R from the equation $x^2 - R = 0$ is :

(A) $x_{i+1} = \frac{x_i}{2}$

(B) $x_{i+1} = \frac{3x_i}{2}$

(C) $x_{i+1} = \frac{1}{2}\left(x_i + \frac{R}{x_i}\right)$

(D) $x_{i+1} = \frac{1}{2}\left(3x_i - \frac{R}{x_i}\right)$

66. Given the two points [a, f(a)], [b, f(b)]. The linear Lagrange polynomial $f_1(x)$ that passes through these points is given :

(A) $f_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{a-b} f(b)$

(B) $f_1(x) = \frac{x}{b-a} f(a) + \frac{x}{b-a} f(b)$

(C) $f_1(x) = f(a) + \frac{f(b)-f(a)}{a-b} f(b)$

(D) $f_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$

67. Consider the following LPP: Maximize $Z = 40x_1 + 88x_2$; subject to $2x_1 + 8x_2 \leq 60$; $5x_1 + 2x_2 \leq 60$; $x_1, x_2 \geq 0$. Then :

(A) $x_1 = 5$ and $x_2 = 10$

(B) $x_1 = 10$ and $x_2 = 5$

(C) $x_1 = 12$ and $x_2 = 4$

(D) $x_1 = 4$ and $x_2 = 12$

68. The general linear programming problem is in standard form if :

(A) The constraints are strict equations

(B) The constraints are inequalities of '≤' type

(C) The constraints are inequalities of '≥' type

(D) The decision variables are unrestricted in sign

69. In a transportation problem, what we must make equal ?

(A) Destinations; sources

(B) Units supplied; units demanded

(C) Columns; rows

(D) Positive cost coefficients; negative cost coefficients

70. If there are n workers and n jobs, then the number of possible assignments is :

(A) (n – 1)

(B) (n – 1)!

(C) n

(D) n!

❖

SPACE FOR ROUGH WORK